电子科技大学
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Online Learning Algorithm and Optimization

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Outline

- Introduction
- Truncated Gradient, FOBOS
- RDA (Regularized dual averaging)
- FTRL (Follow-the-regularized-Leader)
- Discussion and Conclusion

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Part 1 Introduction

Introduction

Predicting CTR & RPM

Introduction

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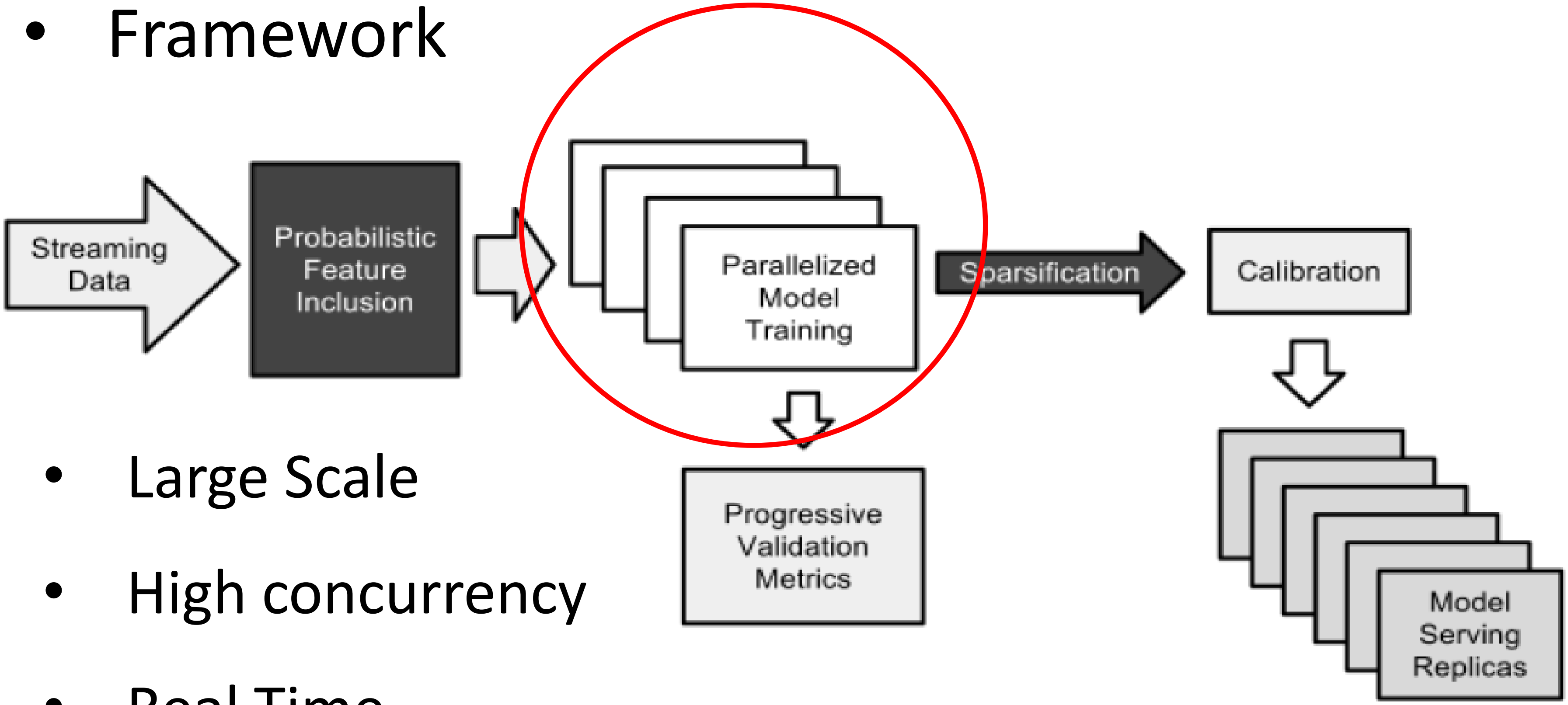
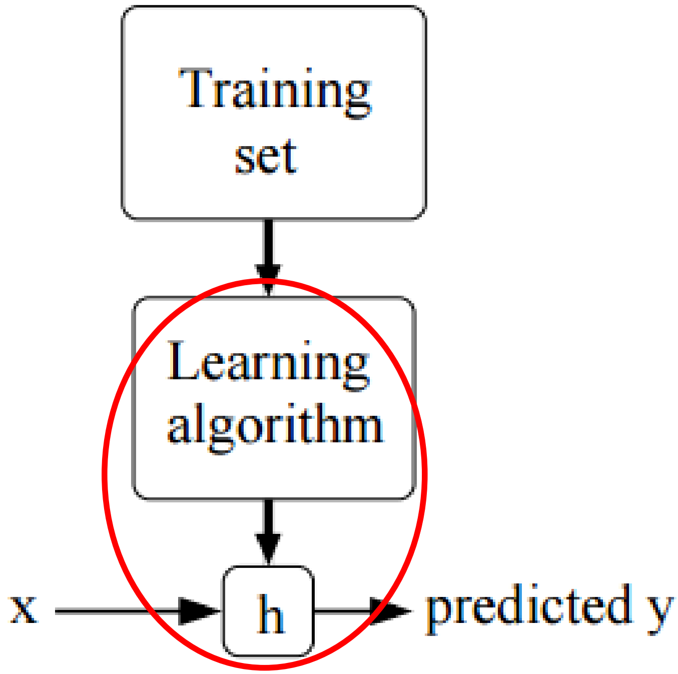
- Framework



- Large Scale
- High concurrency
- Real Time

Introduction

- Framework



* Solving the optimization problem

$$W = arg \min_{W} l(W, Z)$$
$$Z = \{(X_j, y_j) | j = 1,2, \dots, M\}$$
$$y_j = h(W, X_j)$$
$$l(W, Z) = L(W) + \varphi(W)$$

Introduction

- From Batch to Online

Algorithm 1. Batch Gradient Descent

Repeat until convergence {

$$W^{(t+1)} = W^{(t)} - \eta^{(t)} \nabla_W \ell(W^{(t)}, Z)$$

}

Algorithm 2. Stochastic Gradient Descent

Loop {

for j=1 to M {

$$W^{(t+1)} = W^{(t)} - \eta^{(t)} \nabla_W \ell(W^{(t)}, Z_j)$$

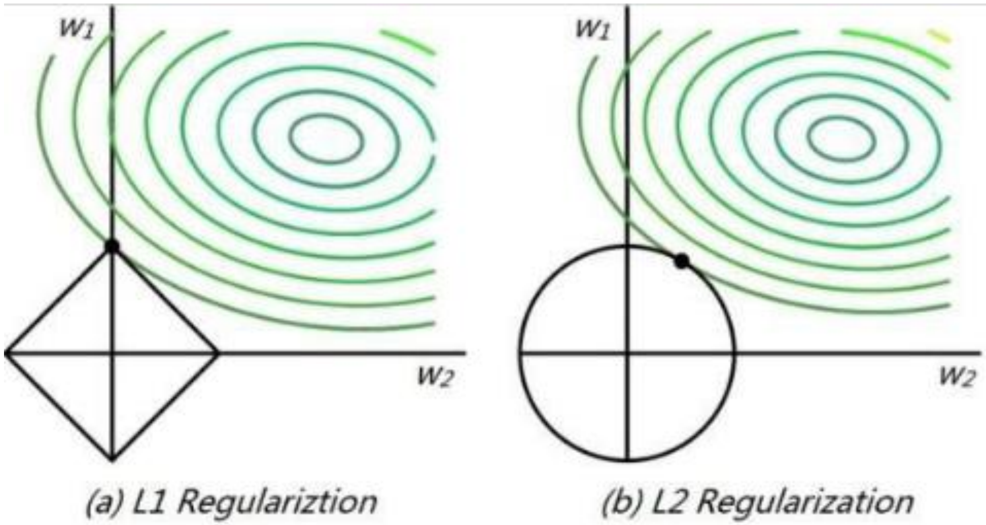
}

}

Part 2 Truncated Gradient, FOBOS

Truncated Gradient

- Sparsity

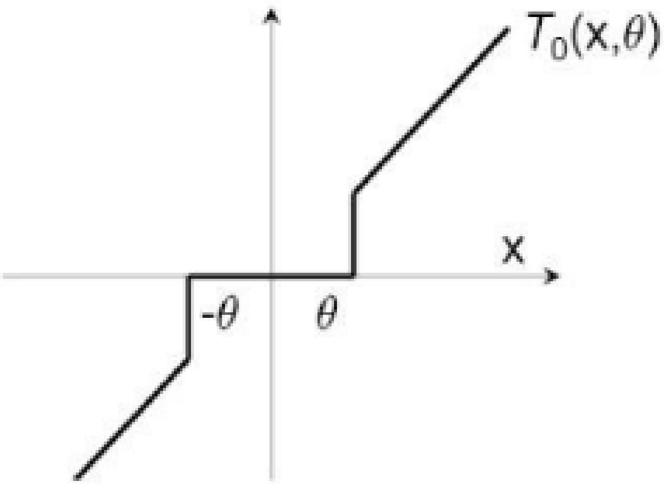


Truncated Gradient

- Aggressive Rounding

$$f(w_i) = T_0(w_i - \eta \nabla_1 L(w_i, z_i), \theta)$$

$$T_0(v_j, \theta) = \begin{cases} 0 & if |v_j| \le \theta \\ v_j & otherwise \end{cases}$$

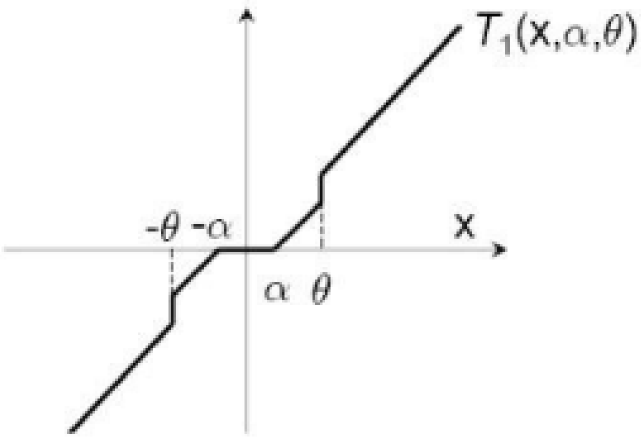


Truncated Gradient

- Smooth rounding

$$f(w_i) = T_1(w_i - \eta \nabla_1 L(w_i, z_i), \eta g_i, \theta)$$

$$T_0(v_j, \theta) = \begin{cases} \max(0, v_j - \alpha) & if\ v_j \in [0, \theta] \\ min(0, v_j + \alpha) & if\ v_j \in [-\theta, 0] \\ v_j & otherwise \end{cases}$$



FOBOS

- Empirical gradient decent and optimization

$$W^{\left(t+\frac{1}{2}\right)} = W^{(t)} - \eta^{(t)} G^{(t)}$$

$$W^{(t+1)} = argmin_W\{\frac{1}{2}\left\|W - W^{\left(t+\frac{1}{2}\right)}\right\|^2 - \eta^{\left(t+\frac{1}{2}\right)} \psi(W)$$

$$W^{(t+1)} = argmin_W\{\frac{1}{2}\left\|W - W^t + \eta^{(t)} G^{(t)}\right\|^2 - \eta^{\left(t+\frac{1}{2}\right)} \psi(W)\}$$

FOBOS

- Iteration

$$if\ W^{(t+1)} = argmin_W F(W)$$

$$0 \in \partial F(W) = W - W^{(t)} + \eta^{(t)} G^{(t)} + \eta^{\left(t+\frac{1}{2}\right)} \partial\psi(W)$$

$$0 = \left\{W - W^{(t)} - \eta^{(t)} G^{(t)} + \eta^{\left(t+\frac{1}{2}\right)} \partial\psi(W)\right\}\Big|_{W=W^{(t+1)}}$$

$$W^{(t+1)} = W^{(t)} - \eta^{(t)} G^{(t)} - \eta^{\left(t+\frac{1}{2}\right)} \partial\psi(W^{(t+1)})$$

FOBOS

- L1-norm

$$Let\ \ \psi(W) = \lambda\|W\|_1, V = [v_1, v_2, \dots, v_N] \in \boldsymbol{R}^N$$

$$W^{(t+1)} = argmin_W \sum_{i=1}^{N} (\frac{1}{2}(w_i - v_i)^2 + \tilde{\lambda}|w_i|\ )$$

$$divided\quad w_i^{t+1} = argmin_W(\frac{1}{2}(w_i - v_i)^2 + \tilde{\lambda}|w_i|\ )$$

FOBOS

- L1-norm

  if w_i^* is the solution then $w_i^* v_i \geq 0$

  because if $w_i^* v_i < 0$

  $\frac{1}{2} v_i^2 < \frac{1}{2} v_i^2 - w_i^* v_i + \frac{1}{2}(w_i^*)^2 < \frac{1}{2}(w_i^* - v_i)^2 + \tilde{\lambda}|w_i^*|$

  for $minimize_{w_i} \left(\frac{1}{2}(w_i - v_i)^2 + \tilde{\lambda}|w_i|\right)$ let $-w_i \leq 0$

  in KKT condition

FOBOS

- Case1 $v_i \geq 0, \omega_i \geq 0$

for $minimize_{w_i}\left(\frac{1}{2}(w_i - v_i)^2 + \tilde{\lambda}|w_i|\right)$

Let β be the Lagrange factor, then use KKT

condition add $-w_i \leq 0$

$$\frac{\partial}{\partial w_i}\left(\frac{1}{2}(w_i - v_i)^2 + \tilde{\lambda} w_i - \beta w_i\right)\Big|_{w_i = w_i^*} = 0 \text{ and } \beta w_i = 0$$

$$\Rightarrow w_i^* = v_i - \tilde{\lambda} + \beta$$

FOBOS

$(1)\ w_i^* > 0:$

$$\beta w_i^* = 0 \Rightarrow \beta = 0$$
$$\Rightarrow w_i^* = v_i - \tilde{\lambda}$$
$$w_i^* > 0 \Rightarrow v_i - \tilde{\lambda} > 0$$

$(2)\ w_i^* = 0:$

$$\Rightarrow v_i - \tilde{\lambda} + \beta = 0$$
$$\beta \geq 0 \Rightarrow v_i - \tilde{\lambda} \leq 0$$

In conclusion $\mathrm{v_i} \geq 0 \Rightarrow w_i^* = \max(0, v_i - \tilde{\lambda})$

FOBOS

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- Case2: $v_i < 0 \quad w_i^* = -\max(0, -v_i - \tilde{\lambda})$
- Conclusion

$$W_i^{(t+1)} = \begin{cases} 0 \quad , & if \ \left|w_i^{(t)} - \eta^{(t)} g_i^{(t)}\right| \leq \eta^{\left(t+\frac{1}{2}\right)}\lambda \\ \left(w_i^{(t)} - \eta^{(t)} g_i^{(t)} - \eta^{\left(t+\frac{1}{2}\right)}\lambda \cdot sgn\left(w_i^{(t)} - \eta^{(t)} g_i^{(t)}\right)\right), otherwise \end{cases}$$

$$\theta = \infty, k = 1, \lambda_{TG}^{(t)} = \eta^{\left(t+\frac{1}{2}\right)}\lambda$$

$$f(w_i) = T_1(w_i - \eta \nabla_1 L(w_i, z_i), \eta g_i, \theta)$$

$$T_0(v_j, \theta) = \begin{cases} \max(0, v_j - \alpha) & if \ v_j \in [0, \theta] \\ min(0, v_j + \alpha) & if \ v_j \in [-\theta, 0] \\ v_j & otherwise \end{cases}$$

Part 3 RDA (Regularized dual averaging)

RDA

- primal-dual algorithmic schema

$$W^{(t+1)} = argmin_W \left\{ \frac{1}{t} \sum_{r=1}^{t} \langle G^{(r)}, W \rangle + \psi(W) + \frac{\beta^{(t)}}{t} h(W) \right\}$$

$$Let \;\; \psi(W) = \lambda \|W\|_1, h(W) = \frac{1}{2} \|W\|_2^2, \{\beta^{(t)} | t \geq 1\}, \beta^{(t)} = \gamma\sqrt{t}$$

$$W^{(t+1)} = argmin_W \left\{ \frac{1}{t} \sum_{r=1}^{t} \langle G^{(r)}, W \rangle + \lambda \|W\|_1 + \frac{\gamma}{2\sqrt{t}} \|W\|_2^2) \right\}$$

RDA

$$W^{(t+1)} = argmin_W \left\{ \frac{1}{t} \sum_{r=1}^{t} \langle G^{(r)}, W \rangle + \lambda \|W\|_1 + \frac{\gamma}{2\sqrt{t}} \|W\|_2^2) \right\}$$

Divided $\Rightarrow$ $$\min_{w_i \in \boldsymbol{R}} \{ \bar{g}_i^{(t)} w_i + \lambda |w_i| + \frac{\gamma}{2\sqrt{t}} w_i^2 \}$$

$$\Rightarrow W_i^{(t+1)} = \begin{cases} 0 \quad , & if \ \left|\bar{g}_i^{(t)}\right| < \lambda \\ \left(-\frac{\sqrt{t}}{\gamma} \left(\bar{g}_i^{(t)} - \lambda \cdot sgn\left(\bar{g}_i^{(t)}\right)\right) \right), & otherwise \end{cases}$$

FOBOS $$\left| w_i^{(t)} - \eta^{(t)} g_i^{(t)} \right| \leq \lambda_{TG}^{(t)} = \eta^{(t+\frac{1}{2})} \lambda \Rightarrow \vartheta\left(\frac{1}{\sqrt{t}}\right) \lambda > \lambda$$

Part 4 FTRL (Follow-the-regularized-Leader)

FTRL

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- The similarity between FOBOS and RDA

$$W^{(t+1)} = argmin_W\{\frac{1}{2}\left\|W - W^t + \eta^{(t)}G^{(t)}\right\|^2 + \eta^{(t)}\lambda\|W\|_1\}$$ --FOBOS

$$\min_{w_i \in \boldsymbol{R}}\{\frac{1}{2}\left(w_i - w_i^{(t)} + \eta^{(t)}g_i^{(t)}\right)^2 + \eta^{(t)}\lambda|w_i|\}$$

$$= \min_{w_i \in \boldsymbol{R}}\{\frac{1}{2}\left(w_i - w_i^{(t)}\right)^2 + \frac{1}{2}\left(\eta^{(t)}g_i^{(t)}\right) + w_i\eta^{(t)}g_i^{(t)} + w_i\eta^{(t)}g_i^{(t)} + \eta^{(t)}\lambda|w_i|\}$$

$$= \min_{w_i \in \boldsymbol{R}}\{w_i g_i^{(t)} + \lambda|w_i| + \frac{1}{2\eta^{(t)}}\left(w_i - w_i^{(t)}\right)^2 + [\frac{\eta^{(t)}}{2}\left(g_i^{(t)}\right)^2 + w_i^{(t)}g_i^{(t)}]\}$$

FTRL

- The similarity between FOBOS and RDA

$$\min_{w_i \in \boldsymbol{R}} \left\{ w_i g_i^{(t)} + \lambda |w_i| + \frac{1}{2\eta^{(t)}} \left(w_i - w_i^{(t)} \right)^2 + \left[\frac{\eta^{(t)}}{2} \left(g_i^{(t)} \right)^2 + w_i^{(t)} g_i^{(t)} \right] \right\}$$

$$\sim \min_{w_i \in \boldsymbol{R}} \left\{ w_i g_i^{(t)} + \lambda |w_i| + \frac{1}{2\eta^{(t)}} \left(w_i - w_i^{(t)} \right)^2 \right\}$$

$$W^{(t+1)} = argmin_W \left\{ G^{(t)} \cdot W + \lambda \|W\|_1 + \frac{1}{2\eta^{(t)}} \|W - W^t\|_2^2 \right\} \text{ L1FOBOS}$$

$$W^{(t+1)} = argmin_W \{ G^{(1:t)} \cdot W + \lambda \|W\|_1 + \frac{1}{2\eta^{(t)}} \|W - 0\|_2^2 \} \text{ L1RDA}$$

FTRL

- The Combination of FOBOS and RDA ---- FTRL

$$W^{(t+1)} = argmin_W \left\{ G^{(1:t)} \cdot W + \lambda_1 \|W\|_1 + \right.$$

FTRL

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DM LESS IS MORE

- Optimization $w_i^* v_i \geq 0, w_i^* \geq 0$

$$W^{(t+1)} = argmin_W \left\{ Z^{(t)} \cdot W + \lambda_1 \|W\|_1 + \frac{1}{2}(\lambda_2 + \textstyle\sum_{s=1}^{t} \sigma^s)\|W\|_2^2 \right\}$$

Divided $\underset{w_i \in \boldsymbol{R}}{minimize} \left\{ z_i^{(t)} \cdot w_i + \lambda_1 |w_i| + \frac{1}{2}\left(\lambda_2 + \sum_{s=1}^{t} \sigma^s\right) w_i^2 \right\}$

$$W_i^{(t+1)} = \begin{cases} 0 \quad , & if \ \left|z_i^{(t)}\right| < \lambda_1 \\ -\left(\lambda_2 + \sum_{s=1}^{t} \sigma^s\right)^{-1} (z_i^{(t)} - \lambda_1 \cdot sgn\left(z_i^{(t)}\right)), otherwise \end{cases}$$

FTRL

Algorithm 1 Per-Coordinate FTRL-Proximal with L_1 and L_2 Regularization for Logistic Regression

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#With per-coordinate learning rates of Eq. (2).
Input: parameters α, β, λ_1, λ_2
$(\forall i \in \{1, \ldots, d\})$, initialize $z_i = 0$ and $n_i = 0$
for $t = 1$ **to** T **do**
  Receive feature vector $\mathbf{x}_t$ and let $I = \{i \mid x_i \neq 0\}$
  For $i \in I$ compute

$$w_{t,i} = \begin{cases} 0 & \text{if } |z_i| \leq \lambda_1 \\ -\left(\frac{\beta+\sqrt{n_i}}{\alpha} + \lambda_2\right)^{-1}(z_i - \text{sgn}(z_i)\lambda_1) & \text{otherwise.} \end{cases}$$

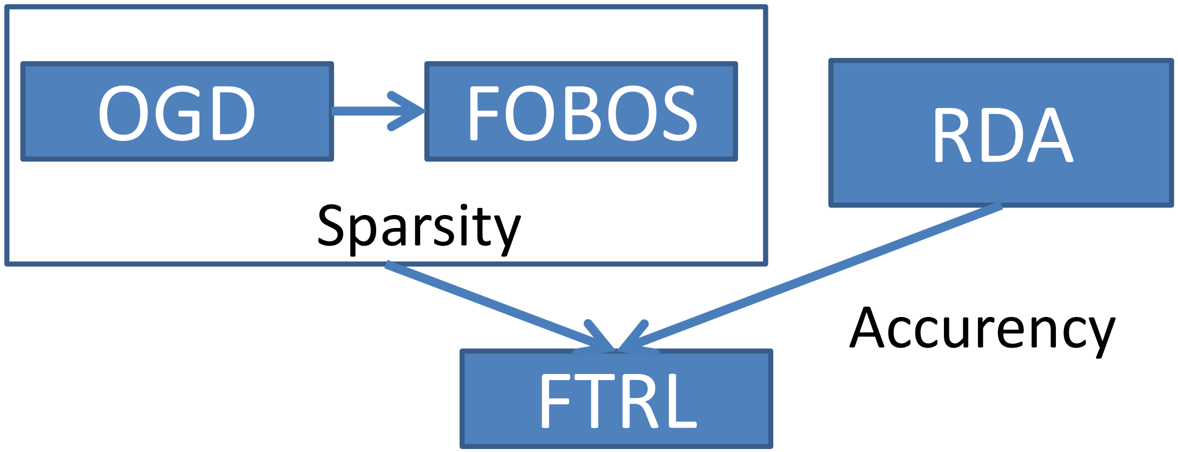
  Predict $p_t = \sigma(\mathbf{x}_t \cdot \mathbf{w})$ using the $w_{t,i}$ computed above
  Observe label $y_t \in \{0, 1\}$
  for all $i \in I$ **do**
    $g_i = (p_t - y_t)x_i$ *#gradient of loss w.r.t.* w_i
    $\sigma_i = \frac{1}{\alpha}\left(\sqrt{n_i + g_i^2} - \sqrt{n_i}\right)$ *#equals* $\frac{1}{\eta_{t,i}} - \frac{1}{\eta_{t-1,i}}$
    $z_i \leftarrow z_i + g_i - \sigma_i w_{t,i}$
    $n_i \leftarrow n_i + g_i^2$
  end for
end for

$$\eta_i^{(t)} = \frac{\alpha}{\beta + \sqrt{\sum_{s=1}^{t}\left(g_i^{(s)}\right)^2}}, \sigma^{(1:t)} = \frac{1}{\eta^{(t)}},$$

$$\sum_{s=1}^{t}\left(\sigma^{(s)}\right) = \frac{1}{\eta_i^{(t)}} = (\beta + \sqrt{\sum_{s=1}^{t}\left(g_i^{(s)}\right)^2})/\alpha$$

Part 5 Discussion and Conclusion

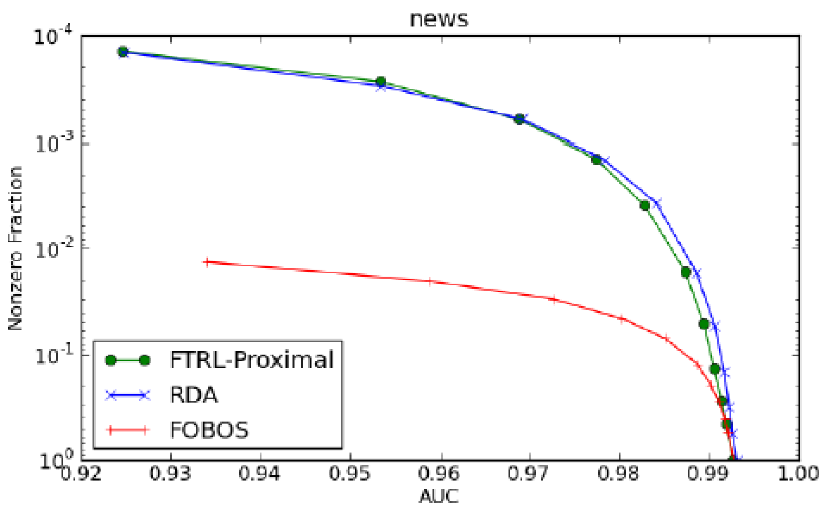
Discussion & Conclusion

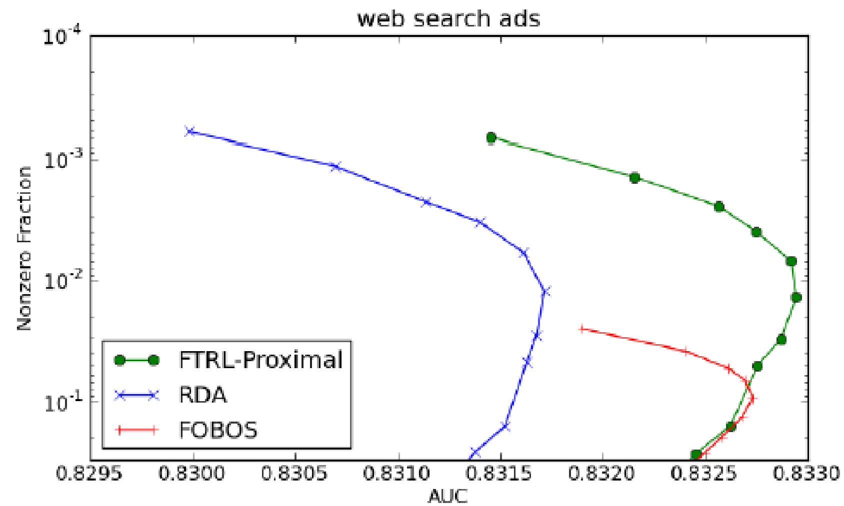


	Num. Non-Zero's	AucLoss Detriment
FTRL-Proximal	baseline	baseline
RDA	+3%	0.6%
FOBOS	+38%	0.0%
OGD-Count	+216%	0.0%

Discussion & Conclusion

DATA	FTRL-PROXIMAL	RDA	FOBOS
BOOKS	0.874 (0.081)	**0.878** (**0.079**)	0.877 (0.382)
DVD	0.884 (0.078)	0.886 (**0.075**)	**0.887** (0.354)
ELECTRONICS	0.916 (0.114)	**0.919** (**0.113**)	0.918 (0.399)
KITCHEN	0.931 (**0.129**)	**0.934** (0.130)	0.933 (0.414)
NEWS	0.989 (**0.052**)	**0.991** (0.054)	0.990 (0.194)
RCV1	0.991 (**0.319**)	**0.991** (0.360)	0.991 (0.488)
WEB SEARCH ADS	**0.832** (**0.615**)	0.831 (0.632)	0.832 (0.849)





Thanks

- Q&A